Comment on "Stable and unstable vector dark solitons of coupled nonlinear Schrödinger equation: Application to two-component Bose-Einstein condensates

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In a recent paper, V. A. Brazhnyi and V. V. Konoto [Phys. Rev. E **72**, 026616 (2005)] investigated the dynamics of vector dark solitons in two-component Bose-Einstein condensates. In the small amplitude limit, they deduced a coupled Korteweg–de Vries equation from the coupled Gross-Pitaevskii equations. They found that two branches of (slow and fast) dark solitons corresponding to the two branches of the sound waves exist. The slow solitons, corresponding to the lower branch of the acoustic wave, appear to be unstable and transform during the evolution into the stable fast solitons (corresponding to the upper branch of the dispersion law). However, our discussion shows that these results are incorrect.

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In Ref. [1], the dynamics of vector dark solitons in two-component Bose-Einstein condensates (BECs) was investigated. In the small-amplitude limit, they deduced a coupled Korteweg–de Vries (KdV) equation from the coupled Gross-Pitaevskii equations. They found that two branches of (slow and fast) dark solitons corresponding to the two branches of the sound waves exist. The slow solitons, corresponding to the lower branch of the acoustic wave, appear to be unstable and transform during the evolution into the stable fast solitons (corresponding to the upper branch of the dispersion law). However, as discussed below, these results are incorrect.

First, in the small-amplitude limit, the KdV equation (18) given by V. A. Brazhnyi and V. V. Konotop [1] is not a coupled KdV equation. According to Eqs. (A1) of Ref. [1], we can obtain the relation

$$q_1 = \frac{2\cos^2\alpha}{v^2 - 2U_1} q_2 \tag{1}$$

and the soliton velocity

$$v = \sqrt{U_1 + U_2 \pm \sqrt{(U_2 - U_1)^2 + \sin^2(2\alpha)}}. \tag{2}$$

Equation (1) means that, in the small-amplitude limit, the two components can be decoupled. Thus, the KdV equation (18) given by Ref. [1] should be a decoupled KdV equation

$$\partial_\tau q_1 + \alpha q_1 \partial_s q_1 + \beta \partial_s^3 q_1 = 0, \tag{3}$$

where $\alpha = 2(\gamma_1^{11} + 2\gamma_1^{12} + \gamma_1^{22})$, $\beta = \beta_1^1 + \beta_1^2$, and $\gamma_1^{11}, \gamma_1^{12}, \gamma_1^{22}, \beta_1^1, \beta_1^2$ are given in Ref. [1]. It is clear that Eq. (3) is a standard KdV equation and has an exact solution of form [2]

$$q_1 = \frac{12\beta}{\alpha}\kappa^2 \operatorname{sech}^2 \kappa(s - 4\kappa^2\tau), \tag{4}$$

where κ is a constant. The second component q_2 can be given by Eq. (1).

Second, the initial conditions given by Eqs. (19)–(23) of Ref. [1] are not correct for the obtained KdV equation; especially for the lower-branch soliton. This can be simply explained as the following. As discussed in Ref. [1], if we set $2\cos^2\alpha = 1 - U_1 + U_2$, then we have $v = \sqrt{U_1 + U_2 \pm 1}$. Thus, for the upper branch ($v = \sqrt{U_1 + U_2 + 1}$), Eq. (1) gives $q_1 = q_2$. However, for the lower branch ($v = \sqrt{U_1 + U_2 - 1}$), Eq. (1) gives

$$q_1 = -\frac{1 - U_1 + U_2}{1 + U_1 - U_2} q_2. \tag{5}$$

Hence, the intial condition given by Eq. (19) of Ref. [1] is an approximate solution of the upper-branch soliton, but not correct for the lower-branch soliton. This results in the instability character of the lower-branch soliton as discussed in Ref. [1] (including their numerical results and the energy analysis in subsection F). That is, the origin of the instability character of the lower-branch soliton observed in Ref. [1] assumes an incorrect initial condition, and is accompanied by quasilinear modes.

Finally, the interesting phenomenon expressed by Eq. (1) is that, for the lower-branch soliton, we can obtain the dark-bright solitons in repulsively interacting two-component BECs [3], i.e., q_1 and q_2 have opposite sign. Because for the lower-branch soliton, Eq. (2) deduces to $v^2 = U_1 + U_2 - \sqrt{(U_2 - U_1)^2 + \sin^2(2\alpha)}$, hence

$$v^2 - 2U_1 = U_2 - U_1 - \sqrt{(U_2 - U_1)^2 + \sin^2(2\alpha)} < 0 \tag{6}$$

for coupled two-component BEC [i.e., $\sin(2\alpha) \neq 0$]. Thus, we have

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$$q_1 = -\frac{2\cos^2\alpha}{\left|\sqrt{(U_2-U_1)^2+\sin^2(2\alpha)}-(U_2-U_1)\right|}q_2. \qquad (7)$$

Equation (7) indicates that the dark-bright solitons in two-component BECs can exist for the lower-branch soliton. This important phenomenon is not given in Ref. [1].

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